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Investigation of Students' Skills in Generating Different Representations to Solve Word Problems: A Case Study in an Elementary School in Indonesia

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Abstract. The purpose of this study is to investigate elementary students' skills in producing different representations in solving mathematical word problems. These skills consist of skills in generating and clarifying ideas and evaluating representations. Data collected in the form of words obtained through interviews, drawings of students' responses to problems, and audio-visual recordings. This study describes the data analysis and interprets the findings using text. A total of 25 elementary students participated in data collection. This study carries out the analysis on the preparation, analysis, exploration, planning, implementation, and verification phases. Results show that students can represent their word problems in different ways and literacy skills. Implications in teaching and learning produce several factors, such as directing students to think creatively. Ultimately, teachers must realize and use students' skills in generating ideas, clarifying ideas, literacy, and evaluating representations in solving word problems in preparing and processing learning.

Keywords: Elementary math education, Students' skills, Verbal representation, Graphical representation, Table representation, Algebraic representation

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INTRODUCTION

Making representations in solving mathematical word problems is not just writing formulas, symbols or operations to produce answers. Students must generate different expressions in describing solutions to problems (Hwang et al., 2007). One of the things students must have to obtain successful solutions in mathematical problem solving is representation skills (Gagné, 1985; Hegarty et al., 1992). Therefore, students should have good representation skills to solve mathematical word problems because if students do not have good representation skills, then the resulting and solution will be incorrect (Sari et al., 2019).

Making proper representations is important in mathematics learning. Abdullah, Zakaria, and Halim (2012) found better learning outcomes and conceptual understanding by applying thinking strategies through visual representation in word problems. Similarly, Cankoy and Özder (2011) stated that the involvement of visual representation in mathematics learning to solve word problems positively affected students' problem-solving performance. Chang, Cromley, and Tran (2016) revealed that assignments require students to provide multiple representations as a solution to improve the quality of mathematics learning. In addition, Sajadi, Amiripour, and Rostamy-Malkhalifeh (2013) demostrated the use of student representation, which will directly impact improving students' problem-solving abilities.

The different ways to generate real representation can be a forum for mathematical thinking (Crespo & Kyriakides, 2007). Representation is the process of modeling concrete things in the real world into abstract concepts or symbols (Hwang et al., 2007; NCTM, 2000). Filloy et al. (2004) proposed that representations are a mental picture of mental development visualized in mathematical thinking. Representation assist students in understanding problems because it can become a media for integrating information that is remembered with previous information (Abdullah et al., 2012). Specifically, in word problems, representation will facilitate students in considering mathematical ideas that are contained in signs, words, symbols, expressions, or pictures (Cankoy & Özder, 2011; Chang et al., 2016; Sajadi et al., 2013). Representation has a role in strengthening students' understanding to build concepts and solve word problems (Stylianou, 2010). Therefore, the representation needed is access to solving word problems.

Some of the previous studies focused on identifying student representations in solving word problems and categorizing the types of representations proposed by students and teachers (Boonen et al., 2014; Özsoy, 2018; Poch et al., 2015). Students who can produce accurate schematic visual representations have better problem-solving abilities than students who cannot (Boonen et al., 2014). Özsoy (2018) found that students with high spatial abilities tend to build schematic structured representations, while students who have low spatial ability tend to build visual representations in solving word problems. In addition, Poch et al. (2015)concluded that visual representations are needed to interpret word problems and when students are solving them. It means that they employ relation processing and have an understanding of interpreting word problems (Boonen et al., 2013). Then, Cromley et al. (2017) formulated the stages of coordinating representation in three main aspects, namely, matching, comparing, and concluding. From the results of several studies above, it can be concluded there has been a gap since no research had described the different representation processes generated by students in solving word problems, which is filled by this research.

Many students have difficulty in representing word problems (Beckmann & Izsák, 2015; Hackenberg, 2010; Ramful, 2014), which are caused by poor problem comprehension and the inability to represent mathematical terms in the word problem. Hackenberg (2010), through "Student and Professor Problems," found that most students mistakenly interpret mathematical terms when understanding word problems. This finding is in line with the investigation of Ramful (2014) in which middle-level students, that is, about 70% of students erroneously constructed word problems for proportional and fractional situations. Moreover, Izsak & Beckmann (2015) uncovered students tend to solve problems that were operational rather than descriptive. The results of previous studies indicate that students still find it difficult to make different representations in solving word problems, and that students tend to only be able to produce one type of representation. Therefore, different representations in solving word problems must be investigated further.

The word problem "5th grade has 28 students and female students are four more than male students. How many female students are in 5th grade?" was given to 5th grade elementary school students. They completed this word problem by generating different representations in solving word problems and finding the right answer. Based on these findings, many elementary school students can make different representations in solving word problems. These findings are different from the research findings described earlier. Similar studies in elementary school students must be investigated further. Researchers suspect that the different representation processes generated by elementary students have a role in solving word problems. In addition, previous studies failed to discuss the ability of elementary students to produce different representations in solving word problems. Therefore, this study aims to investigate elementary students' skills in producing different representations in solving mathematical word problems. The skills intended to produce different representations comprises skills of of generating ideas, clarifying ideas and evaluating (Swartz & Perkins, 1990). Thus, the results of this study can contribute as one of the literatures for educators and researchers to know how to shape cognitive abilities and processes from the start so that students can have different representations in solving word problems and also directing the achievement of learning and teaching that are oriented to student creativity.

METHODS

Research Design

This research used a qualitative approach with a case study design. This study determined that students learn about solving word problems, either in class or in additional coursework classes. Ensuring that prospective subjects have the knowledge and experience in solving mathematical word problems is necessary.

Participants

This study involved 25 5th grade elementary school students in Makassar, Indonesia. Students were asked to complete word problems given by researchers. Students' answers were grouped on the basis of the resulting representations, namely, algebraic representations and correct answers. Algebraic representations but incorrect answers and different representations are finding correct answers. The two subjects chosen in this study generated different representations, namely, (verbal-graph-algebraic) and (verbal-table-algebraic) for in-depth interviews related to the various representations generated. Students were chosen on the basis of good communication skills, confidence, and willingness to participate in the research. The teacher helped researchers to choose students who met these requirements. The researchers conducted interviews to closely track the different representations generated.

Data collection tools and Procedure

The instruments used in this study were taken from *Mathematical Problem Solving*, the yearbook of the association of mathematics educators (Kaur et al., 2009). The researchers translated the

> Class 4A has 3 more girls than Class 4B.
>
> Each class has 15 boys.
>
> Write down three math questions about the two classes.
>
> You can involve other numbers if needed.

Figure 1. *Task of mathematical problem posing*

word problems into Indonesian. This word problem was used as a task for problem posing in Grades 3 and 5 of elementary schools in Singapore. **Figure 1** presents these word problems.

Some considerations for adopting the problem-posing problem into this research are the problems presented in the study are equally used for elementary school students. Other than that, the school-level curriculum in Singapore is related to the problem of the level of the elementary school curriculum in Indonesia is still at the level of problem solving. Therefore, the problem is simple from a mathematical problem posing to complicated word mathematical problems. Then, in addition to changing the scores of many male and female students, it was adjusted to the student data at the study site. It was made to be more contextual to make it easier for students to solve word problems, and can attract students' attention to solve word problems in **Figure 2**.

This study set the research procedure as follows. First, we classically assigned word problems to Grade 5 elementary school students, and then gave these tasks with the approval of the

> 5th grade has 28 students and female students are four more than male students. How many female students are in 5th grade?

Figure 2. *Adaptation of word problems in this study*

class teacher. Second, we identified and classified students' answers on the basis of the representation generated in solving word problems. Third, we conducted interviews with students regarding the resulting representations. Fourth, we analyzed the data of the task results and student interviews. Fifth, we made audio visual records of the student interview process and transcribed it afterward.

Analysis of data

This study used a qualitative data analysis for the data analysis technique, which is carried out exploratively and continuously and requires continuous reflection until the data is saturated. The data analysis steps are, processing and preparing the data, reading the entire data, analyzing in more detail and coding, describing data, presenting the data in narrative form, and interpreting the data (Creswell, 2017).

RESULTS

The researchers gave word problem to 25 elementary school students as the initial activity, then asked the students to solve the word problems according to the specified time. The researchers collected, identified, and grouped the answers on the basis of representation generated in solving word problems. The 25 students who solved the word problem obtained a grouping of 56% of students used algebraic representations and produced correct answers, 28% of students used algebraic representations but produced incorrect answers, and 16% of students produced different representations and correct answers. However, in this study, the researchers focus on two-student work that develops different representations (verbal-graph-algebraic) and (verbal-table-algebraic) in solving word problems. Here are the results of solving word problems and subject interviews using different representations with the coding of Subjects 1 (S1) and 2 (S2) then described to be understood and interpreted.

Description of Skills Subjects (S1) Generate Different Representations

Different representations generated by S1 (verbal-graph-algebraic). Results on researcher interviews at S1. The first thing that S1 did after obtaining a word problem was reading it multiple times. The goal is to determine the meaning of each sentence in the text word problem and understand the problem. Next, S1 began solving the word problem by writing and marking the subject "male = L" in black ink and "female = P" in red ink. The reason S1 used two different inks was to be easily distinguished.

Verbal Representation

The first representation generated by S1 in solving word problems was verbal, accompanied by $P = L + 4$. The researcher question "Why should it be written down again, why not directly draw the graph," S1 answered, "This paper (while pointing to a verbal representation), as a guide for the next representation. Then the researcher asked again "Why graph pictures, why not circles, tables or anything else." S1 answered, "Because I used to use graphs when answering questions, and if graphs. Easy to draw to compare things."

Graph Representation

The second representation generated by S1 arises when the researchers asked "compare something that how" S1 answered, "as in this word problem, because there are four women more than men, the female bar picture is higher." **Figure 3** shows the graphical representation by S1.

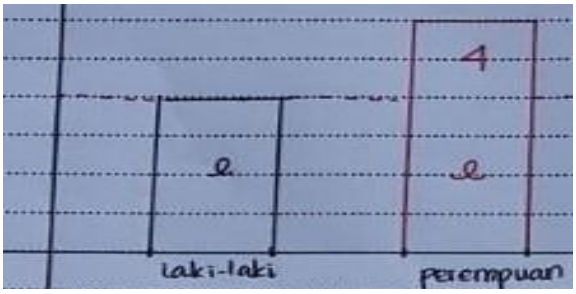

Figure 3. *Graph representation by S1*

The second representation generated by S1 must be present, because the embodiment of the verbal representation of S1, and the two are interrelated. Hence, both of them must be shown. From the researcher's question, "If you did not draw this graph, what do you think?" S1 answered, "It cannot be absent, because it is explained in words this (while pointing to verbal representations), is interrelated." Researchers again asked the use of two different colored writings, and S1 answered: "to distinguish men and women." The researcher did not ask for the presence of the symbol L on the black trunk and the L with four on the red trunk because S1 explained at the beginning of the interview. In addition, S1 utilized the equation generated by S1 in verbal representations and graph representations to be used in algebraic representation until finding the correct answer.

Algebraic Representation

The last representation generated by S1 in solving the word problem to find the correct answer is an algebraic representation in **Figure 4**

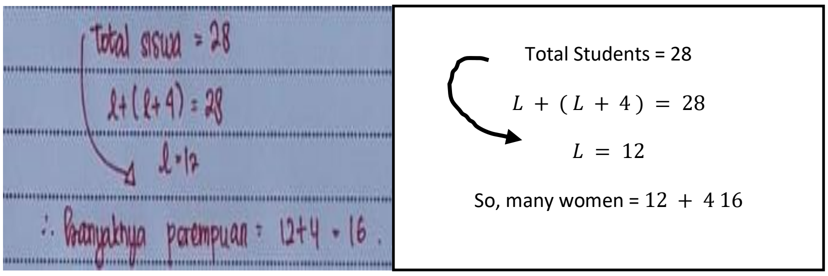

Figure 4 . *S1 Algebraic representation to find the right answer*

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Based on a brief S1 algebraic representation, the researcher considers that S1 significantly understood the word problem. Evidenced by the results of interviews of several researchers' questions related to S1 algebraic representation.

R: Why is the answer short?
S1: Yes, I already know the process because I used to work on problems like this
R: What questions do you usually answer?
S1: My desire because I often participate in the Math Olympiad, both at national and international levels
R: Where did it come from $L + (L + 4) = 28$*?*
S1: From here (while pointing to the representation of Figure 4) $\cdots$ *from* $L + (L + 4) = 28$*, then I grouped the same into* $(L + L) + 4 = 18$*. So we get* $2L + 4 = 28$*. Furthermore,* $2L + 4 - 4 = 28 - 4$*, becomes* $2L = 24$*, then* $L = 12$
R: Are you sure about your answer?
S1: Yes, uh ... wait for a check again (students only need about 10 seconds and say" yes, I am sure that the answers I found were correct

Description of Skills Subjects (S2) Generate Different Representations

The presentations generated S2 representations, namely, (verbal-table-algebraic). Based on researcher interviews at S2, the first thing S2 did after obtaining the word problem was similar to the activities carried out by S1, which was repeatedly reading to determine the meaning of each sentence in the text word problem to understand the problem in the word problem. Other S2 activities are identical to those by S1, namely, writing, for example, the subject "male = L" and "female = P." Reasons for S2 to be easy to distinguish.

Verbal Representation

The first representation generated by S2 was a verbal representation. Researchers questioned "Why to start by writing" checkered method because the number of women is four more than men, then the table is 2 x 2 as follows. The researcher questioned while showing a verbal representation of S2 "Why start with writing this? S2 paused for a moment and answered, "To explain the table picture." The researcher questioned again "I mean, why was not the table drawn directly? or was not answered immediately in order to find the answer?" S2 answered, "Because I want my answer to be different from other friends." A recurring question from the researcher "But why do not just draw the table?" alternatively, answer it directly by counting?" S1 fell silent again and answered, "Sorry because I want to draw first before calculating the answer to the word problem." The researchers question again "But why boxes or tables? Why not circles or graphs? S1 answered, "Because I like the pictures of boxes and tables, and I want to illustrate the meaning of, 'Four women are more than men based on the text word problem'."

Table Representation

The second representation produced by S2 arises when researchers dig deeper information by asking the question "Then why are there $2 \, x \, 2$?" S2 answered, "Because the table I made is two columns and two rows." **Figure 5** presents the table representation by S2

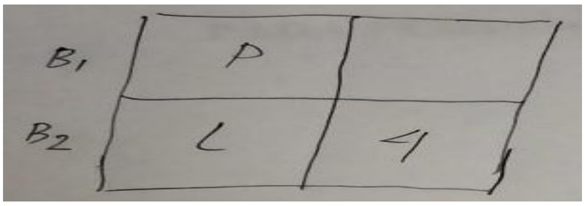

Figure 5. *Table representation by S2*

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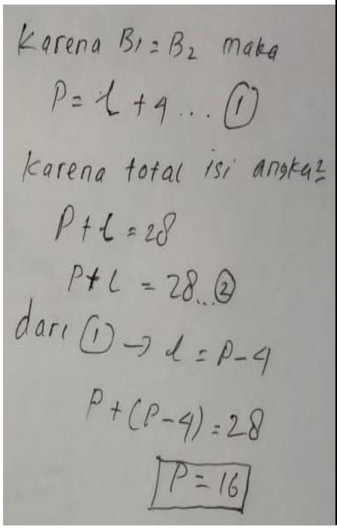
The reason S2 is the same as S1 about the need for (second) table representation. S2 revealed that the first representation (verbal) and the second representation (table) are interrelated. When table representation is not, then representation is incomplete. It was revealed by S2 when the researcher question: “What if you do not make a table?” S2 paused for a moment and answered, “If I did not draw this picture (pointing to **Figure 5**), then this does not apply (pointing to **Figure 4**) because the two are related.” The researcher also asked about the existence of the symbols B1 and B2 written by S2 on the left side of the table and the symbols P, L and number 4 in **Figure 5**. S2 answered, “B1 = first row is P (female) and B2 = line the second is L (male), while the number 4 in B2 indicates that women are equal to many men. L added four according to the text word problem that says (four women more than men).”

Algebraic Representation

The last representation generated by S2 is an algebraic representation. S2 utilized the text word problem, which was understood to produce two equations, namely, "P = L + 4 ... (1)” as the first equation and “$P + L = 28$,” as the second equation. S2 revealed that the source "$P = L + 4 \ldots (1)$,” because it would be counting “four women more than men.” Meanwhile, $P + L = 28$,” because “Many of the students in the classroom are 28.” This equation was used in algebraic representations to find the correct answer. **Figure 6** shows the algebraic representation.



Because $B_1 = B_2$, then

$P = L + 4 \cdots\cdots (1)$

Because of the total contents the numbers

$P + L = 28$

$p + l = 28 \cdots (2)$

From(1) $\rightarrow l = p - 4$

$p + (p - 4) = 28$

$p = 16$

Figure 6. *Algebraic Representation S2 and find answers*

S2 algebraic representation on solving word problems makes it clear that the given word problem was significantly understood. S2 proves by generating different and interrelated representations to find the right answer. This evidence presented through a research interview. The researcher's question is related to the representation"$B1 = B2$.” S2 answered, "$B1 = B2$ because of 2 x 2, two columns of two rows. While the origin of "$L = P$ – 4," the researcher did not question again because it was written in the answer, that it came from Equation 1 in **Figure 8.** So the researcher immediately questioned the source of "$P + (P - 4) = 28$.” S2 answered, “From here (pointing to Equation 1) and from here (pointing to Equation 2).” The next research question "Why from $P + (P - 4) = 28$ you immediately wrote $P = 16$? How do getting it?” S2 immediately answered quickly and said that “From $P + (P - 4) = 28$, keep opening the brackets so that $P + P - 4 = 28$, then becomes $2P - 4 = 28$, then $2P - 4 = 28$, then respectively - each segment is added 4, so $2P = 28 + 4$, becomes $2P = 32$, then 32 divided by 2, the result is $P = 16$." So many female students are 16. Apart from the test-based interview results, the researcher also conducted interviews with S2 on the basis of the movement or style of the subject. The researchers' final question, about the ups and downs in a series of movements and while pointing to the answer sheet when answering the researcher's questions, and after the subject found the answer to the word problem. S2 answered, "If that is the case, I look back after the answer that I am working. Other than, want to know whether process and answers are correct or not."

DISCUSSION and CONCLUSIONS

The use of representation is important in solving word problems. Students represent problem solving means doing the process of modeling concrete things in the real world into abstract concepts or symbols (Hwang et al., 2007). Representation has a role in strengthening students' understanding to build concepts and solving problems especially in word problems (Stylianou, 2010). Then, representation serves as a medium to assist students in understanding and integrating recalled information with new information presented in the problem (Abdullah et al., 2012). Furthermore, representation reduces the difficulty in solving word problems, for example in communicating ideas between signs, words, symbols, expressions, or pictures (Cankoy & Özder, 2011; Chang et al., 2016; Sajadi et al., 2013). Therefore, representation is essential in solving word problems.

This study aims to investigate the skills of students in producing different representations in solving mathematical word problems by giving tests to 25 elementary school students in the city of Makassar, Indonesia. Then choosing two subjects to be interviewed in-depth on the basis of the different representations generated, namely, S1 (verbal-graph-algebraic) representation, S2 representation (verbal-table-algebraic), and finding the correct answer. Swartz and Perkins (1990) revealed the different representation skills of students can occur if students have the skills in managing information of a problem. The information is processed mentally to become a representation with different types. The management of this information requires creative-thinking abilities of students. Meanwhile, to determine the accuracy of the representation raised, Murtafiah, Sa'dijah, Chandra and Susiswo (2019) pointed out that this requires the accuracy of decision making, which is part of critical thinking skills. As'ari, Kurniat, and Subanji (2019) explained that most students tend to produce only one representation and this can be minimized by teaching mathematics especially at the elementary school level to not only focus on the accuracy and correctness of the answers.

Based on the results of the analysis of the different representations generated by students. Found three skills, namely, generating ideas, clarifying ideas, and evaluating. Generating ideas is when students express that skills produce different representations in solving word problems are supported by students' prior knowledge. Clarifying ideas show that students can explain and describe the reasons for producing different representations one-by-one. Ultimately, the evaluation skills show when they are silent for a while before answering the researchers' questions. Students double-check every representation generate and re-check the overall representation that results in solving word problems.

Student skills generate ideas. Students express that the different representations generated by students in solving word problems are supported by students' prior knowledge. Generating ideas is shown by presenting three types of representations, namely, (verbal-graphic-algebra) which as interrelated in a series. Costa (2001) and Kirkwood (2005) found that in making decisions related to the knowledge a person can consider several choices through brainstorming, predicting consequences, and making the "best" choices by weighing the good and the bad. Then Türnüklü and Yeşildere (2007) found that to teach mathematics effectively, teachers must have a deep understanding of mathematical knowledge about the material they teach. Swartz and Perkins (1990) suggested that generating ideas, retention and use of information skills is part of creative-thinking skills that refers to skills encouraging learning for understanding and use of active knowledge, and critical thinking skills refer to skills to assess the reasonableness of ideas. Titikusumawati, Sa'dijah, As'ari, and Susanto (2019) unveiled that students who had an average level of academic skills began to develop their creative thinking skills. Murtafiah et al. (2019) also Swartz and Perkins (1990) revealed that generating ideas, clarifying ideas, and assessing the reasonableness of ideas is part of skills in decision making in solving mathematical problems

Students' skills in clarifying ideas show that students can explain and describe one-by-one; the reasons and responses produce different representations. For example, S1 generated the first representation (verbal) and its purpose as a hint to illustrate the second representation (graph). Then the presentation of the second representation (graph) S1 because it was specifically instructed on the first representation (verbal), namely, instructions to describe the graph. It was the third representation (algebra) produced by S1 as a computational process to find the right answer. Mishra and Koehler (2006) argued that teachers who have a good understanding of the subject matter can clarify ideas to produce different ways that are accessible to students. An, Kulm, and Wu (2004) revealed that to calibrate ideas, a strong knowledge base is essential in the domain of knowing mathematical concepts that are usually difficult for students to understand, including about misconceptions and knowing the sources of student mistakes. Swartz and Perkins (1990) revealed that the skill of clarifying ideas is part of decision making based on students' knowledge. Then, Murtafiah, Wasilatul, Sa'dijah, and Chandra (2018) pointed out that descriptive explanations, giving reasons, and interpretations were the things most often done by students to clarify answers.

The skill of evaluating was revealed by students that when I paused for a moment before answering the researcher's questions. Means recheck every representation generated and re-check the overall representation generated after solving the word problem is complete and find answers. Swartz and Perkins (1990) revealed that decision making in the problem-solving process must be re-evaluated to find out whether our decisions are suitable or not. Dindyal, Tay, Toh, Leong, and Quek (2012) found that after solving the world problem, checking it again was important. The aim is to convince us by solving the resulting problem.

In addition to students having the skills of generating ideas, clarifying ideas and evaluating, students have literacy skills. Literacy skills are expressed by students in developing different representations on the basis experiences and habits of solving math Olympics questions that are similar to this word problem. Smith, Hardman, and Higgins (2006) found that mathematics literacy is substantially helpful for teachers and students in the learning process in the classroom. Literacy strategies can influence traditional patterns of student interaction throughout the class (Amir et al., 2019; Smith et al., 2004). In addition, Beard (2000) found that literature reading in students during class helps students in learning and offers a significant promise for the improvement of the living standards and opportunities of thousands of children.

This study found three types of skills in producing different representations in solving mathematical word problems, namely, generating ideas, clarifying ideas, and evaluating. Other than that, other skills possessed by students were found in this study, namely, literacy skills. Students' literacy skills in producing different representations can in subsequent studies or other researchers.

The relevance of this research to the teaching and learning of mathematics produces several things, such as directing students to think creatively. Moreover, teachers should realize that students' skills in generating ideas, clarifying ideas, and evaluating skills affect the results of solving word problems. This should be taken into consideration in preparing learning plans and should be used in the learning process in the classroom, especially in solving word problems. It should be introduced earlier at the elementary school level by using simple problems and be a reference for other researchers to study the other representation processes.

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